Modeling of the Gross Regional Product on the Basis of Production Functions

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ABSTRACT

The article is devoted to elaboration and construction of a static model of macroeconomics in which economics is considered as an unstructured holistic unit, the input of which receives the resources, and the output is the result of the functioning of economics in the form of gross domestic product or gross regional product. Resources are considered as arguments, and gross output – as function. Simulation is carried out using the production function for Russian Federation in general and for its eighty-three regions and eight federal districts. For building the models there were selected such macroeconomic indicators as gross regional product, value of fixed production assets, population, number of people employed in the economy, number of economically active population. For each region the model was built in current and comparable prices, with and without allowance for technological progress. Macroeconomic models used statistical data for 15 years (2000 – 2014) and the number of built models is 2208. The appropriate software "EGRMod" was engineered to work under Windows operating systems using MS Access or under control of Access Runtime library. External Access database is used to collect data. Numerical calculations are performed in SQL language using VBA.

Introduction

Russian Federation has the world's largest territorial extent that along with its advantages creates a number of problems associated with the effectiveness of management of functioning and development at the regional level. Therefore, economic-mathematical analysis of the socio-economic development of regions and its prediction is a very relevant task, which allows to determine the direction of economic development of Russian Federation.

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Successful socio-economic development of the region is impossible without effective management of the regional economics. In turn to improve the efficiency of management of regional economics it is important to use economic-mathematical modeling and forecasting of its development, which allows you to make cost-based decisions that promote optimal economic management strategies. On the way to achieving this goal, it seems appropriate to build mathematical models of forming the gross regional product.

Regional differences in socio-economic development in Russian Federation are very significant. Analysis of its' development suggests that over time differences in the level of production and consumption in different regions is not only not declining, but tends to increase. Differentiation between subsidized and prosperous regions is increasing more and more.

Currently, the identification of the most efficient methods of evaluation of socio-economic development is the main goal of research in this area. Searching methods of the evaluation was primarily due to the opportunity to determine the various options of the development of the region with the greatest accuracy and lowest cost. Comparative analysis of growth rates of different regions is interesting for state regulation of regional development. This comparison will allow to take timely decision to support a particular region in one or another form. Unfortunately, comparative assessment of regional structures is a complex process, due to the fact that integral indicators used for the analysis often do not allow a realistic comparison.

Research methods

The purpose of the study is to build a static macroeconomic model of the regional economy on the basis of production functions Cobb-Douglas, multiplicative production functions with (and without) technological progress, in current and comparable prices for the Russian Federation and all the regions and Federal districts of the Russian Federation.

Research methods:

– theoretical research, i.e. the analysis of macroeconomic models based on production functions for the application to solve the set tasks; the learning approaches, methods and methodology in regional modelling and forecasting;

– empirical: collection, analysis, and initial processing of necessary statistical data; regression analysis using applications developed in SQL using VBA.

Methodological Framework

The task of analysis and assessment of socio-economic development of regions is a subject of study in economics for quite some time (Tsaregorodtsev & Sajranova, 2015; Malykh, Polyanskaya & Lebedev, 2015; Dayneko, 2011). Currently not only the researchers in regional economics consider this issue, but many other experts, which include economists-mathematicians, use different mathematical methods and models (Abakumov, Krylov & Antoshchuk, 2000; Grishin, 2010; Klochko, Fomenko & Nekrasova, 2016), ranging from models of interindustry balance and ending with the systems of econometric equations and models (Tsaregorodsev & Sarycheva, 2009; Timirgaleeva & Grishin, 2013; Mosunova & Tsaregorodtsev, 2006), to solve this problem. In practice, there are different techniques and methods of assessment of the level of regional

development with the application of mathematical methods. In this research, we consider methods based on the usage of econometric models, whose parameters allow a quantitative assessment of the level of regional development (Antipova et al., 2016).

The most commonly used indicators include: gross regional product (or its growth rate, the volume of investments into fixed capital, share of employed in economics and share of economically active population, percentage of minimum subsistence level for average per capita income, share of population with incomes below the subsistence minimum, level of education. In assessing the level of socio-economic development of the region the traditional indicators that determine the level of production and consumption of goods (gross regional product (GRP), nominal and real GRP per capita, growth rate of these indicators) also cannot be underestimated.

Currently, researchers of regional development pay particular attention to the innovative component of areas, as there is a direct link between the innovativeness of the region and its socio-economic development (Kokotkina & Sadovin & Kokotkina, 2014; Becker, 2003; Gurban & Myzin, 2011). The modern development of the economic system is due to the creation and introduction into production the latest achievements of science and technology (Kulalaeva, Kreneva & Kanyugin, 2016; Tsaregorodtsev, Semagin & Mosunova, 2009). Innovation is the key to improve the competitiveness of economics, economics based on knowledge. Innovation policy is an integral part of the strategy of industrial-innovative development and is the main tool of increasing the competitiveness of economics.

Economic-mathematical models, of the form:

$$Y = F(K, L, H, R, T),$$
(1)

which is based on production function (PF), form a separate group of single-loop macroeconomic models. Here the volume of production (Y) depends on physical capital (K), population (L), human capital (health and education etc., H), resources – land, raw materials, etc. (R), level of technological development (T).

As a rule, most of these quantities are difficult, if at all possible, to be quantified. Therefore, the models based on production functions, are mainly theoretical and consider the effects of changes in any quantitatively modifiable factors (physical capital, labor) related to the dynamics of population.

Regardless of the form of production function, such models have a number of distinctive features. First of all, describing the influence of factors on volume of output (value added, gross output, value of production, etc.), the production function assumes a relatively free mutual substitution of these factors. Secondly, the production function implies that increasing any of the factors is automatically accompanied by increasing other factors or at least one of them. Thirdly, the production function is homogeneous, which means that at simultaneous increase of all factors, the resulting (endogenous) variable is incremented in a strictly defined proportional to the increase of factors. Fourth, such models employ the hypothesis of stable population in which the rate of population growth is equal to the growth rate of population of working age.

In macroeconomics PF can be used to describe the relationship between the annual cost of resources and the annual ultimate production of products across the region or country. The production system here is the region or the country as a whole. PF are based on statistical data and are mainly used for solving tasks of analysis, planning and forecasting (Tsaregorodtsev & Sajranova, 2015).

Resources at the macro level often regard accumulated labor in the form of productive assets (capital K) and real (live) work (L), and as a function of gross production (Y). Capital and gross output are measured quantitatively in value terms (current and comparative), work – in real terms, using official statistics. Then macroeconomics is modeled by the following nonlinear macroeconomic PF:

$$Y = F(K, L) \tag{2}$$

The simplest production function that reflects the impact of two factors of production – labor and capital, expressed in the form of Cobb-Douglas production function $Y = \alpha_0 K^\alpha L^{1-\alpha}$, where $\alpha_0 > 0$, $\alpha > 0$, and α_0 – the coefficient characterizing the level of performance. A more sophisticated model is represented by a multiplicative production function of the form $Y = \alpha_0 K^{\alpha_1} L^{\alpha_2}$, where $\alpha_1 > 0$, $\alpha_2 > 0$. They also consider linear (additive) production functions, for example, of the form $y = a_0 + a_1 x_1 + a_2 x_2$.

The transition from additive PF to multiplicative PF is done using logarithms. So the multiplicative function becomes additive:

$$\ln Y = \ln \alpha_0 + \alpha_1 \ln K + \alpha_2 \ln L, \tag{3}$$

and back in the potentiation.

Often, the production function can also include a description of the technical progress (TP) as a function of time $A(t)$ (Nikolaeva et al., 2015). TP is affected by either the efficiency of a particular resource (in this case, output grows at a fixed physical volume of this factor) or total output. In these cases, we have:

$$\text{1)} \quad Y_t = F(K_t, L_t \cdot A(t)) \tag{4}$$

increasing the productivity of capital – capital energy TP, or TP according to (Harrod, 1973).

$$\text{2)} \quad Y_t = F(K_t \cdot A(t), L_t) \tag{5}$$

productivity increasing – labor-saving TP, or TP according to (Solow, 1974) .

$$\text{3)} \quad Y_t = F(K_t, L_t) \cdot A(t) \tag{6}$$

growing total factor productivity – neutral TP, or TP for Hicks (Aukucionek 1984). If the rate of neutral TP γ is constant, then $A(t) = e^{\gamma \cdot t}$. Then, for example, a multiplier of PF, taking into account exogenous technical progress can be represented as:

$$Y_t = e^{\gamma\, t} F(K_t, L_t) = \alpha_0 e^{\gamma\, t} K_t^{\alpha_1} L_t^{\alpha_2} ,$$ (7)

where $e^{\gamma\, t}$ take into account the impact of scientific and technological progress (STP), $\gamma > 0$ characterizes the rate of growth of production under the influence of NTP, and α_1 equal the elasticity of output by fixed assets, and α_2 – output elasticity for labor. If $\alpha_1 > \alpha_2$, there is a labor-saving (intensive) growth, otherwise vandeberghe (extensive) growth.

Usually when you move to n – measured ($n > 2$) PF as an additional argument (resource) is administered the volume of used natural resources (Kokotkina et al., 2015). Then the relevant production function, taking into account the effect of natural resources R may take the form of:

$$Y_t = e^{\gamma\, t} F(K_t, L_t) = \alpha_0 e^{\gamma\, t} K_t^{\alpha_1} L_t^{\alpha_2} R^{\alpha_3}$$ (8)

The PF parameters are determined, generally speaking, the method of least squares for time series issues and resources (Y_t, K_t, L_t), where

$$Y_t = Y(t),\ K_t = K(t),\ L_t = L(t), \text{ and } t = \overline{0, T} \text{ with a step } \Delta t = 1,$$
T – length of the time series, t – number of the year.

This assumes that we have the following ratio:

$$Y_t = \delta_t \alpha_0 K_t^{\alpha_1} L_t^{\alpha_2} ,$$ (9)

where δ_t – corrective random factor, resulting in the conformity of the actual and calculated releases, and reflect the variation of the result under the influence of factors not considered with the expectation $E\delta_t = 1$. This ratio represents the model of multiple linear regression.

Multiplicative PF was first evaluated by Cobb and Douglas in 1928 for the US economy. As examples can give some calculations on the economy:

a) USSR for 1960-1985.:

$$Y = 1{,}022 K^{0{,}538} L^{0{,}467};$$ (10)

b) Russian Federation for 1980-1994.:

$$Y = 0{,}931 K^{0{,}539} L^{0{,}594};$$ (11)

c) USA for 1960-1995.

$$Y = 2,248K^{0,404}L^{0,803}. \tag{12}$$

Production function $Y = F(K, L)$ can have the following properties:

1. $F(0,0) = F(0,L) = F(K,0) = 0$ (13)

2. $\dfrac{\partial F}{\partial K} > 0, \ \dfrac{\partial F}{\partial L} > 0$ (14)

3. $\dfrac{\partial^2 F}{\partial K^2} \leq 0, \ \dfrac{\partial^2 F}{\partial L^2} \leq 0.$ (15)

3.1. $\dfrac{\partial^2 F}{\partial K \partial L} \geq 0.$ (16)

4. $F(+\infty, L) = F(K, +\infty) = +\infty.$ (17)

5. $F(tK, tL) = t^p \cdot F(K, L).$ (18)

Property 1 indicates that in the absence of at least one of the resources, production is impossible.

Property 2 means that with the increase in the cost of at least one of resources, volume of production increases.

Property 3 means that an increase in resources, the rate of production slows down. And condition 3.1 means that the growth of one of the resources, limiting the effectiveness of another resource increases.

Property 4 means that with unlimited increase of one of the resources, production is growing indefinitely.

Property 5 means that PF is a homogeneous function of degree (order)

$$p > 0.$$

If $p > 1$, with the growth of the scale of production in t times, production

volume increases in t^p times. That is, the productivity gain from economies of scale (increasing returns to scale).

If $p < 1$, so we have falling production efficiency of scale (waning impact of scale)

If $p = 1$, we have a constancy of production efficiency due to increased scale (constant returns to scale).

It is easy to verify by direct calculation that the PF Cobb-Douglas satisfies all the properties $1 - 5$, and is characterized by constant returns to scale.

In the study of growth factors of the economy allocate extensive factors of growth by increasing the resource cost by increasing the scale of production, the intensive factors of growth by improving resource efficiency. In order to highlight these factors with the help PF move on to the relative (unitless)

indicators, as the problem of comparing present and past work satisfactorily in economic theory is still not solved.

Suppose that in some base year PF has the form:

$$Y_0 = \alpha_0 K_0^{\alpha_1} L_0^{\alpha_2}. \tag{19}$$

Then the transition from the PF to a multiplicative dimensionless form can be done as follows:

$$\frac{Y}{Y_0} = \frac{\alpha_0 K^{\alpha_1} L^{\alpha_2}}{\alpha_0 K_0^{\alpha_1} L_0^{\alpha_2}} = \left(\frac{K}{K_0}\right)^{\alpha_1} \left(\frac{L}{L_0}\right)^{\alpha_2}. \tag{20}$$

If enter designations

$$\widetilde{Y} = \frac{Y}{Y_0}, \quad \widetilde{K} = \frac{K}{K_0}, \quad \widetilde{L} = \frac{L}{L_0}, \tag{21}$$

then PF will look like:

$$\widetilde{Y} = \widetilde{K}^{\alpha_1} \widetilde{L}^{\alpha_2}. \tag{22}$$

From the dimensionless form can easily switch to PF multiplier:

$$Y = \frac{Y_0}{K_0^{\alpha_1} L_0^{\alpha_2}} K^{\alpha_1} L^{\alpha_2} = \alpha_0 K^{\alpha_1} L^{\alpha_2}, \tag{23}$$

where the coefficient

$$\alpha_0 = \frac{Y_0}{K_0^{\alpha_1} L_0^{\alpha_2}} \tag{24}$$

resources commensurate with the issue.

Let us now define the so-called generalized indicator of the efficiency of the economy by defining two private pre-performance indicator:

$$\widetilde{A}_K = \frac{\widetilde{Y}}{\widetilde{K}} - \text{capital productivity} \quad \text{and} \quad \widetilde{A}_L = \frac{\widetilde{Y}}{\widetilde{L}} - \text{productivity}.$$

Then the generalized indicator of economic efficiency equal to the average geometric partial indicators of efficiency

$$E = \sqrt[\alpha_1 + \alpha_2]{\widetilde{A}_K^{\alpha_1} \cdot \widetilde{A}_L^{\alpha_2}} = \widetilde{A}_K^{\alpha} \cdot \widetilde{A}_L^{1-\alpha}, \tag{25}$$

where

$$\alpha = \frac{\alpha_1}{\alpha_1 + \alpha_2} \text{ и } 1 - \alpha = \frac{\alpha_2}{\alpha_1 + \alpha_2}.$$

As the scale of production of M is evident in the amount of wasted resource,

$$M = \widetilde{K}^{\alpha} \cdot \widetilde{Y}^{1-\alpha}, \tag{26}$$

and the total output will be calculated as the product of efficiency E on the scale of production M:

$$\widetilde{Y} = E \cdot M. \tag{27}$$

Let Y_t, K_t, L_t – appropriate volumes of production and resources at the time t. Going to the next moment of time $(t+1)$, can consider the rate of growth of output Y_{t+1}/Y_t, which is:

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{K_{t+1}}{K_t}\right)^{\alpha_1} \cdot \left(\frac{L_{t+1}}{L_t}\right)^{\alpha_2}, \tag{28}$$

or

$$\left(\frac{Y_{t+1}}{Y_t}\right)^{\frac{1}{\alpha_1 + \alpha_2}} = \left(\frac{K_{t+1}}{K_t}\right)^{\alpha} \cdot \left(\frac{L_{t+1}}{L_t}\right)^{1-\alpha}, \tag{29}$$

where α and $1 - \alpha$ – the relative elasticity of production factors From the last equality it follows that if then the issue is growing faster than average grow factors, but if $\alpha_1 + \alpha_2 < 1$ - then slower. . Indeed, for example, when $\alpha_1 + \alpha_2 > 1$:

$$\frac{Y_{t+1}}{Y_t} > \left(\frac{Y_{t+1}}{Y_t}\right)^{\frac{1}{\alpha_1+\alpha_2}} = \left(\frac{K_{t+1}}{K_t}\right)^{\alpha} \cdot \left(\frac{L_{t+1}}{L_t}\right)^{1-\alpha}, \tag{30}$$

that is, the rate of growth of output is greater than the average growth rate of factors. Thus, if the elasticity of production $\alpha_1 + \alpha_2 > 1$, then PF describes a growing economy.

Let us now consider the growth rate of production in discrete form:

$$y_t = \frac{Y_{t+1} - Y_t}{Y_t}, \ \ k_t = \frac{K_{t+1} - K_t}{K_t}, \ \ l_t = \frac{L_{t+1} - L_t}{L_t},$$

in a continuous form:

$$y_t = \frac{Y'_t}{Y_t}, \ \ k_t = \frac{K'_t}{K_t}, \ \ l_t = \frac{L'_t}{L_t}.$$

Then PF $Y = F(K,L)$ in the voluminous records can be presented in a so-called tempo records

$$y = f(k,l). \tag{31}$$

Consider, for example, a multiplier of PF of the form

$$Y_t = e^{\gamma t} F(K_t, L_t) = \alpha_0 e^{\gamma t} K_t^{\alpha_1} L_t^{\alpha_2}, \tag{32}$$

where $e^{\gamma t}$ take into account the impact of scientific and technological progress, $\gamma > 0$ characterizes the rate of growth of production under the influence of NTP. Lets prelogarithmic this function:

$$\ln Y_t = \ln a_0 + \gamma t + \alpha_1 \ln K_t + \alpha_2 \ln L_t, \tag{33}$$

and differentiate with respect to t:

$$\frac{dY_t}{Y_t} = \gamma \, dt + \alpha_1 \frac{dK_t}{K_t} + \alpha_2 \frac{dL_t}{L_t}, \tag{34}$$

or

$$\frac{Y'_t}{Y_t} = \gamma + \alpha_1 \frac{K'_t}{K_t} + \alpha_2 \frac{L'_t}{L_t}. \tag{35}$$

Thus, a multiplier in PF volume indicators corresponds to a linear relationship for continuous growth

$$y_t = \gamma + \alpha_1 k_t + \alpha_2 l_t.$$ (36)

If we consider a discrete growth rate, taking into account the approximate equality $\Delta Y_t \approx dY_t$, obtained in this equation, an approximate equality. That is, in the discrete case, bulk PF corresponds to a linear formula when the growth rate $y,\ k,\ l$.

Note, however, that these formulas (equations) are equivalent only in the continuous case. The statistical assessment of parameters $\alpha_1,\ \alpha_2,\ \gamma$ of these equations apply to the discrete sample data. Therefore, the evaluation of $\widehat{\alpha}_1,\ \widehat{\alpha}_2,\ \widehat{\gamma}$, obtained, for example, for a nonlinear equation, it is incorrect to carry on linear equation and Vice versa.. Even if we evaluate these equations on the same observations, you can get totally different value assessments. And one of the resulting regression equations can be significant and the other not significant.

This means that one way of estimation (e.g., non-linear equation could have a meaningful statistical result, and the other (e.g., linear equation). Therefore, it is better to estimate both these equations, and if it is obtained similar statistically significant results, then this will serve as the confirmation of compliance with the evaluated formula the real relationships of variables.

Note that from the linear equation, it follows that the constant γ представляет represents the rate of neutral technical progress that is not associated with the growth of labor l_t and capital k_t, and reflects the intensification of production at the macro level.

When you use linear equations for practical calculations, one should also take into account the fact that the parameter is constructed as a residual. In other words, it reflects the influence on the rate of growth of output all other factors except labor and capital. This means that this parameter characterizes the impact on output and some other factors not considered. However, for most macroeconomic processes, the dominant role among the "other factors" belongs to technical progress.

In the economic analysis of the constructed regression models of the form $Y = F(K, L)$ of great importance is the elasticity of substitution of factors of production. For example, the elasticity of substitution (substitution) of labor by capital is the amount of:

$$\sigma_K = \sigma_{L,K} = \frac{d\left(\dfrac{K}{L}\right)}{dS_K} \bigg/ \frac{\dfrac{K}{L}}{S_K} = \frac{d\left(\dfrac{K}{L}\right)}{dS_K} \cdot \frac{S_K}{K/L},$$ (37)

where $S_K = -\dfrac{dK}{dL}$ – the marginal rate of substitution of labor by capital.

Value σ_K shows how many per cent will change the capital-labor ratio to labor K/L when changing the marginal rate of substitution of labor by capital by one percent. In other words, σ_K characterizes the degree of substitutability of factors.

Similarly entered and the elasticity of substitution of labor funds

$$\sigma_L = \sigma_{K,L} = \frac{d\left(\dfrac{L}{K}\right)}{dS_L} \Bigg/ \frac{\dfrac{L}{K}}{S_L} = \frac{d\left(\dfrac{L}{K}\right)}{dS_L} \cdot \frac{S_L}{L/K}, \tag{38}$$

where $S_L = -\dfrac{dL}{dK}$ – the marginal rate of replacement of capital by labor.

We calculate these parameters for the Cobb-Douglas production function. As

$$S_K = \frac{\alpha_2}{\alpha_1}\frac{K}{L}, \quad \text{то} \quad \frac{K}{L} = \frac{\alpha_1}{\alpha_2}S_K,$$

and

$$\frac{d(K/L)}{dS_K} = \frac{\alpha_1}{\alpha_2}. \tag{39}$$

Then,

$$\sigma_K = \frac{\alpha_1}{\alpha_2}\frac{\dfrac{\alpha_2}{\alpha_1}\dfrac{K}{L}}{\dfrac{K}{L}} = 1 = \sigma_L. \tag{40}$$

However, the initial hypothesis of unit substitutability of the factors of production can serve as a specific limitation in the practical use of multiplicative production functions for economic analysis of macroeconomic processes. Therefore, and are considered appropriate extensions of the class of production functions. So the most famous generalization of the production function is Cobb-Douglas production function is CES (constant elasticity of substitution) function with constant elasticity of substitution ($\sigma_K = \sigma = const$), which can be represented as:

$$Y = F(K,L) = A\left[\alpha K^{-\rho} + (1-\alpha)\cdot L^{-\rho}\right]^{-\frac{1}{\rho}}, \tag{41}$$

where $\rho = \dfrac{1-\sigma}{\sigma}$.

As an example, the evaluation of PF CES we give some estimates for the economy of the former USSR. Such estimates were made for different periods range 1950-1987. For Example, (Kolemaev, 1998) gives the following estimate for the Soviet economy over the 1960-1985:

$$Y = 0,966 \cdot \left(0,4074 \cdot K^{-3.03} + 0,5926 \cdot L^{-3.03}\right)^{-\frac{1}{3,03}} \cdot e^{0,0252\,t},$$

(42)

where $R^2 = 0,9982, \quad DW = 1,76$.

From the point of view of the obtained values of coefficient of determination and statistics of Durbin-Watson, this relationship is statistically significant. The elasticity of substitution

$$\sigma = \frac{1}{1+\rho} = \frac{1}{1+3,03} = 0,25.$$

(43)

The estimates of the elasticity of substitution obtained by other investigators, is also less than unity: 0.4 - (Sadovin & Kokotkina, 2014), 0.37 to 0.43 for different periods. (Sadovin, 2010), 0,37-0,40 (Easterly & Fisher, 1995). In General, we can conclude that the elasticity of substitution for the economy of the USSR was approximately 0.4 mm. This indicates low substitutability of labor and capital. This value was much lower than in PF Cobb-Douglas, where the pre-assumed equal to one. Therefore, the fallacy of the initial hypotheses regarding the degree of interchangeability of factors may cause the statistical insignificance of the estimates of PF Cobb-Douglas

In addition, some economists (Easterly & Fisher, 1995) believe that the low level of substitutability of labor and capital was one of the main reasons of stagnation of the Soviet economy. At a low elasticity of substitution of excess capital accumulation does not provide the expected growth of output, and work with consistently low performance became a limiting factor in the growth of the economy.

Data, Analysis, and Results

The structure of the studied data includes observations of the same economic units that were implemented in different moments of time (2000 – 2014). The sample combines data on the spatial type (cross-section), and the data type of time series (time-series). Thus, at each moment of time there is evidence of spatial type on the gross regional product, and for each unit the appropriate data form (short) time series. Therefore, the evaluation was performed with the panel data structure. For the construction of static macroeconomic models of regions of the Russian Federation with the use of production functions was used statistical data for 15 years (2000 – 2014) through 83 regions and 8 Federal districts, as well as in the whole of the Russian Federation (Federal state statistics service 2016) . The appropriate software "EGRMod" designed to work under Windows operating systems using

MS Access or under control of library AccessRuntime. To collect data using an external Access database. Numerical calculations are performed in SQL using VBA.

AS main characteristics of the model, we used the following variables:

1) gross regional product – Y;

2) the value of fixed assets – K;

3) the population – L_1;

4) the number of economically active population – L_2;

5) the number of people employed in the economy – L_3;

For each region the model was built in current and comparable prices (adjusted to consumer price index) in the form of the production function is Cobb-Douglas $Y = \alpha_0 K^\alpha L^{1-\alpha}$, the production function is Cobb-Douglas with technical progress $Y = \alpha_0 K^\alpha L^{1-\alpha} e^{\gamma \cdot t}$, the multiplicative production function $Y = \alpha_0 K^{\alpha_1} L^{\alpha_2}$ and, given technological progress – $Y = \alpha_0 K^{\alpha_1} L^{\alpha_2} e^{\gamma \cdot t}$. For each function considered three types of labor resources L_1, L_2, L_3. Thus, it was built 2208 models for all regions and Federal districts, with the exception of the Crimea and Sevastopol, due to problems related to the lack of reliable statistical data.

Imagine some of the constructed models.

1) Far Eastern Federal district:

$$Y = 103{,}2760 \cdot K^{-0{,}1820} \cdot L_1^{1{,}1820} \cdot e^{0{,}0868 \cdot t}$$ – in comparable prices;

$$Y = 7{,}7888 \cdot K^{-0{,}1551} \cdot L_2^{1{,}5459} \cdot e^{0{,}0813 \cdot t}$$ – in comparable prices;.

2) Volga Federal district:

$$Y = 0{,}0743 \cdot K^{1{,}2493} \cdot L_3^{-0{,}2493}$$ – at current prices;

$$Y = 43{,}0441 \cdot K^{0{,}0713} \cdot L_2^{0{,}9287} \cdot e^{0{,}0570 \cdot t}$$ – in comparable prices;.

3) Siberian Federal district:

$$Y = 1{,}7276 \cdot K^{0{,}6926} \cdot L_3^{0{,}3074} \cdot e^{0{,}0697 \cdot t}$$ – at current prices;

$$Y = 0{,}1043 \cdot K^{1{,}2395} \cdot L_2^{-0{,}2582} \cdot e^{0{,}0010 \cdot t}$$ – at current prices.

4) the North-West Federal district:

$$Y = 0,1362 \cdot K^{1,1843} \cdot L_2^{-0,1843} \text{ – in comparable prices;}$$

$$Y = 3102,4154 \cdot K^{-0,6846} \cdot L_2^{1,6846} \cdot e^{0,1066 \cdot t} \text{ – in comparable}$$

prices;

5) Ural Federal district:

$$Y = 0,5175 \cdot K^{0,9368} \cdot L_2^{0,0632} \text{ – at current prices;}$$

$$Y = 0,8113 \cdot K^{0,8661} \cdot L_2^{0,1339} \cdot e^{0,0116 \cdot t} \text{ – at current prices.}$$

6) Central Federal district:

$$Y = 0,4981 \cdot K^{0,9909} \cdot L_1^{0,0091} \text{ – at current prices;}$$

$$Y = 2,5189 \cdot K^{0,6917} \cdot L_3^{0,3083} \cdot e^{0,0172 \cdot t} \text{ in comparable prices;.}$$

7) South Federal district:

$$Y = 0,0629 \cdot K^{1,2770} \cdot L_3^{-0,2770} \text{ – at current prices;}$$

$$Y = 0,1734 \cdot K^{1,0848} \cdot L_3^{-0,0848} \cdot e^{0,0259 \cdot t} \text{ – at current prices.}$$

8) North Caucasian Federal district:

$$Y = 0,1039 \cdot K^{0,7756} \cdot L_2^{0,5310} \cdot e^{0,0437 \cdot t} \text{ – at current prices;}$$

$$Y = 0,0020 \cdot K^{1,1950} \cdot L_2^{0,2920} \cdot e^{0,00131 \cdot t} \text{ – in comparable prices.}$$

9) Russian Federation:

$$Y = 0,0782 \cdot K^{1,2745} \cdot L_3^{-0,2745} \text{ – in comparable prices;;}$$

$$Y = 88,3065 \cdot K^{-0,0195} \cdot L_2^{1,0195} \cdot e^{0,0680 \cdot t} \text{ in comparable}$$

prices;.

The constructed model can be further used for constructing appropriate macroeconomic projections, for example, using a dynamic model of economic growth by R. Solow (1974), and solving problems of optimal management of investment processes at the level of regions of the Russian Federation.

Discussion and Conclusion

Practical application of models based on production function was limited due to problems of quantitative measurement of some key parameters of these models. This predetermines some and the ambiguity of the conclusions based on such models. This ambiguity stems from the fact that at constant economies of scale in production and constant share of labor in total population level of performance becomes dependent on the presence of additional factors and technology. Hence the increase in population leads to slower productivity growth, if population growth will not affect the development of other factors of

production and/or technology. If the population growth will weaken the development of other factors of production and/or technology, labor productivity growth will slow even more. If population growth stimulates the development of other factors of production and/or technology, labor productivity growth will accelerate or slow down depending on the ratio of the level of influence of positive and negative effects.

However, models based on production functions, it is theoretically confirmed the existence of a relationship between growth in population and growth rate of produced goods per capita, making the rationale for calculations of correlation between these indicators. Another thing that is forced ignoring the impact of other key factors of economic growth can often lead to diametrically opposite conclusions and the low statistical significance of the results.

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